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*Tech Talk Presents*

Modeling the Range  
Performance of the Electra 10E  
– Amelia Earhart's Aircraft



## Modeling the Range Performance of the Electra 10E – Amelia Earhart’s Aircraft

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### About TIGHAR

The white paper presented here was developed in cooperation with The International Group for Historic Aircraft Recovery (TIGHAR). TIGHAR is a non-profit foundation dedicated to promoting responsible aviation archaeology and historic preservation. It has been involved since 1988 in investigating Amelia Earhart’s last flight. More information about TIGHAR and the Earhart Project can be found on the web at <http://www.tighar.org>.

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# Modeling the Range Performance of the Electra 10E – Amelia Earhart’s Aircraft

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## Abstract

A computer model is developed to simulate the flight time, distance and fuel usage of the Lockheed Electra 10E aircraft under arbitrary conditions and pilot inputs. The model is based on engineering data contained in Lockheed’s Maximum Range Report prepared by Kelly Johnson and W. C. Nelson in 1936. These data are used to develop a set of differential equations which are then integrated to compute the aircraft’s state at regular intervals during the flight. The results of the computer model are compared with the Lockheed’s original calculations and are found to be in agreement.

## Introduction

A persistent question in the history of aeronautics is the fate of Amelia Earhart. Her attempt to fly around the world in an Electra 10E failed when she was unable to reach Howland Island on July 2, 1937. The fact that her flight disappeared leaving little or no evidence to support an investigation has led to wide ranging speculation on the circumstances of her disappearance.

Many of the competing theories ignore the capabilities of Earhart’s aircraft, the Electra 10E. Others give extreme significance to particular technical specifications that are difficult to know with certainty. In order to illuminate this field of investigation, a computer model has been developed to simulate the performance of the Electra 10E. The model is faithful to the engineering data and computation methods used by Kelly Johnson and W.C Nelson of Lockheed in their range study developed for this aircraft.

Within this paper, we will first develop a mathematical model derived from the report. Then we will describe how the performance model is embodied in a computer program. With the program developed, then, we are able to verify it by comparing its results with the report.

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### From Lockheed Report to Mathematical Model

Our first task, then, is to develop an accurate mathematical model of the Electra 10E. This is achieved by studying Lockheed Report 487 and extracting the necessary information.

### Lockheed’s Maximum Range Report

Lockheed Report 487 is an engineering report prepared by Kelly Johnson and W.C. Nelson of Lockheed in June of 1936. The report was evidently prepared to convince Amelia Earhart that the Electra 10E was suitable for her upcoming World Flight. In the report Mr. Johnson estimates that the aircraft has a maximum range of 4100-4500 miles and a flight time of 26-30 hours. The report goes on to provide recommended operating procedures for maximum range.

In an extensive appendix to the report, W.C. Nelson documents the method of computation for the range estimates in the report proper. The appendix contains all of the engineering data, formulas and procedures for evaluating the aircraft’s performance for an arbitrary flight profile. Between the appendix and the report itself, there is sufficient information to develop the model we seek.

This section is intended to serve as a reader’s guide to Lockheed’s report 487. It points out where particular information can be found in the report and how it can be interpreted. A copy of report 487 can be found on the web at [http://tighar.org/Projects/Earhart/Documents/Report\\_487/Report487.pdf](http://tighar.org/Projects/Earhart/Documents/Report_487/Report487.pdf).

### Constants

First, a number of constants required for the model are contained within the report.

Constant	Value	Units	Page
Wing Area	458.3	Square feet	p. 30
Propeller Diameter	9.0	feet	Derived from middle table on p. 16
Max Takeoff Weight	16500	pounds	p. 4 and other locations
Empty Weight	9300	pounds	p.12 graph and other locations
Fuel Volume	1200	gallons	p. 4 and other locations
Fuel Weight	7200	pounds	derived from p. 12 and other locations
Fuel Density	6.0	pounds / gallon	derived from above.
max $C_L$	1.46	none	p. 21



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Most of these quantities are mentioned explicitly or can be read directly from the various graphs. One that requires some explanation, however, is the propeller diameter. This value doesn’t appear directly in the paper. But on page 16, the middle table is a worksheet for computing thrust power for various airspeeds at sea level, 600 BHP engine power, 2300 RPM. The second column,  $V/nD$  is a function of airspeed,  $v$ , prop speed,  $n$ , and prop diameter,  $D$ . Of these  $v$  and  $n$  are given and  $D$  is the value we seek. So we can solve for  $D$  using any one (or all) of the rows of this table. From the first row we have:

$$V/nD = 90 \text{ [mph]} / (2300 \text{ [RPM]} * D) = .382$$

Rearranging terms and converting 90 mph to 7920 ft/min we get

$$D = 7920 \text{ [ft/min]} / (2300 \text{ [RPM]} * .382)$$

or

$$D = 9.01 \text{ [feet]}$$

### Lift, Drag and Thrust Required

Next, we seek formulas for lift and drag. Page 30 of the report shows a worksheet for computing lift, drag and thrust power required for flight at sea level with no flaps. This page also presents data for coefficient of lift,  $CL$ , coefficient of drag,  $CD$ , and Horsepower required,  $HP_{req}$ . It is worth noting that this horsepower figure represents thrust horsepower,  $THP$ , not brake horsepower,  $BHP$ . Hence, this is the output power of the propellers, not the engines themselves.  $CL$  and  $HP_{req}$  appear in formulas on the bottom part of the page. Both are given in terms of  $q$ , the dynamic pressure, which means we can evaluate them for any air density and airspeed, not just sea level flight. (A more detailed discussion of  $q$  appears below.) What is missing from page 30 is a formula for  $CD$ , the coefficient of drag. Fortunately, an expression for  $CD$ , as a function of  $CL$ , appears on page 24. Also missing on page 30 is the maximum  $CL$  the wing can provide before the stall condition manifests itself. But this value can be determined from the expression on page 21,  $0.9 * CL_{max} = 1.31$ . This gives a  $CL_{max}$  of 1.46.

The expressions for  $CD$  and  $CL_{max}$  given above apply for the flaps retracted configuration. The values of  $CL_{max}$  can be determined for other flap configurations from page 22. And partial graphs for  $CD$  for other flap configurations appear on page 24 but it isn’t clear that complete formulas for  $CD$  can be derived for those configurations. Initially, we will only model the flaps retracted configuration.

### Dynamic pressure, $q$

All computations of lift and drag begin by computing the dynamic pressure of the air stream. Dynamic pressure,  $q$ , is a function of airspeed and density. It is defined as

$$q = \frac{1}{2} \rho v^2$$

Values of  $q$  for various velocities at sea level are given on page 30. While the report does not explicitly contain this formula for  $q$ , the use of this formula and the letter  $q$  to denote dynamic pressure are



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common in the field of aerodynamics. And it is easy to confirm that, given an appropriate value of  $\rho$  for air density at sea level, the values in the table on page 30 are reproduced by the formula. Units of  $q$  in the report are pounds / square foot. (Note that units of  $q$  on page 30 are given as percentages [%]. This appears to be a transcription error. It should be #/ft<sup>2</sup>. The same values of  $q$  also appear in the top table on page 16. Here the units are correctly transcribed.)

### **Coefficient of Lift, $C_L$**

The lift generated by the aircraft’s wing is described by the equation,

$$Lift = C_L A q$$

where  $C_L$  is the coefficient of lift,  $A$  is the area of the wing and  $q$  is the dynamic pressure, described above. For the case where the aircraft is in level flight, we know the lift is equal to the aircraft’s gross weight,  $W$ , and we can solve for  $C_L$ ,

$$C_L = \frac{W}{Aq}$$

(from page 30). The aircraft can vary  $C_L$  by changing the wing’s angle of attack but the maximum  $C_L$  that can be attained with flaps up is  $C_{Lmax} = 1.46$  (page 21). Greater angles of attack would lead to a stall condition.

### **Coefficient of Drag, $C_D$**

The aircraft’s overall drag is represented by  $C_D$ , the coefficient of drag. Page 24 contains a formula for the drag coefficient as a function of the lift coefficient:

$$C_D = 0.029 + \frac{C_L^2}{20.75 e}$$

where

$$e = 0.85 - 0.0667 \times C_L$$

Using these expressions, we can determine  $C_D$  from a known  $C_L$ . And from there we can determine the whole drag force from the expression

$$Drag = C_D A q$$

(page 30).

### **Power Required**

Finally, we are on the threshold of determining the thrust power required by the Electra 10E under any arbitrary flight conditions. Power is defined as



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$$Power = Force \times Velocity.$$

or , applying this to our situation with drag being the force and airspeed the velocity:

$$P_{thrust} = C_D A q v.$$

For A in square feet, q in pounds per square foot and v in feet per second, the units of power are foot-pounds per second. With the appropriate conversion factor to horsepower and substituting the constant wing area for A, we get

$$P_{thrust} = 1.22 C_D q v_{mph}$$

as on page 30 of the report.

### Propeller Efficiency and Available Thrust

The propellers of the Electra 10E convert engine output power (BHP) into thrust horsepower (THP). How much thrust was produced is a function of not just engine output but air density, propeller speed and airspeed. And the Electra 10E had a variable-pitch propeller with a constant-RPM controller. So, the propeller’s performance also depended on the propeller pitch. The pitch itself is also a function of density, propeller speed and airspeed.

The Lockheed report does not describe the procedure for computing the propeller efficiency for a given set of parameters. It does, however, contain a reference to NACA T.R. 351 where the relevant computational procedure is described. And the Lockheed report contains the curves necessary to carry out the procedure. Finally, the report contains tables of results for specific combinations of parameters. From these we can determine the computation procedure and verify it.

The characterization of the propeller is done in terms of two dimensionless quantities, Cs and V/nD.

### Cs

Cs is a dimensionless coefficient relating air density ( $\rho$ ), airspeed (v), engine power (P) and propeller speed ( $\omega$ ). The formula defining Cs is:

$$Cs = \frac{\rho v^5}{P \omega^2}$$

[p 397 of NACA TR-351.] Where the input parameters have the following units:





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Symbol	Meaning	Units
$\rho$	Air density	Slugs per cubic foot
$v$	Airspeed	Feet per second
$P$	Engine power	Foot-pounds per second
$\omega$	Propeller speed	Revolutions per second

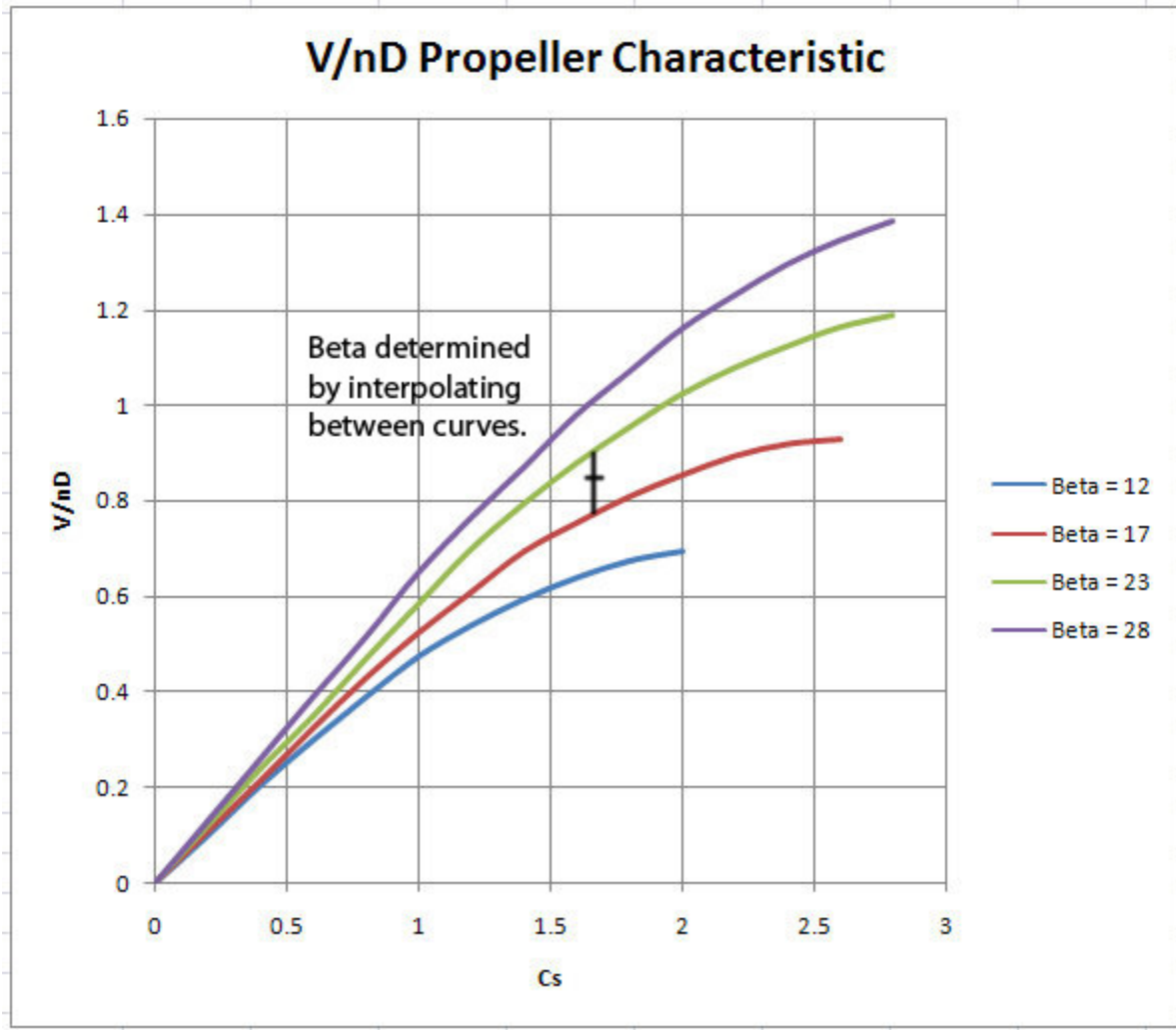
### **V/nD**

$V/nD$  is the second dimensionless quantity used to characterize the propeller’s performance. It is the ratio of  $V$ , forward progress (airspeed) to propeller tip speed,  $nD$ .  $V/nD$  is computed by simply converting airspeed ( $V$ ) to feet per minute and dividing by prop speed in rpm ( $n$ ) times prop diameter in feet( $D$ ).



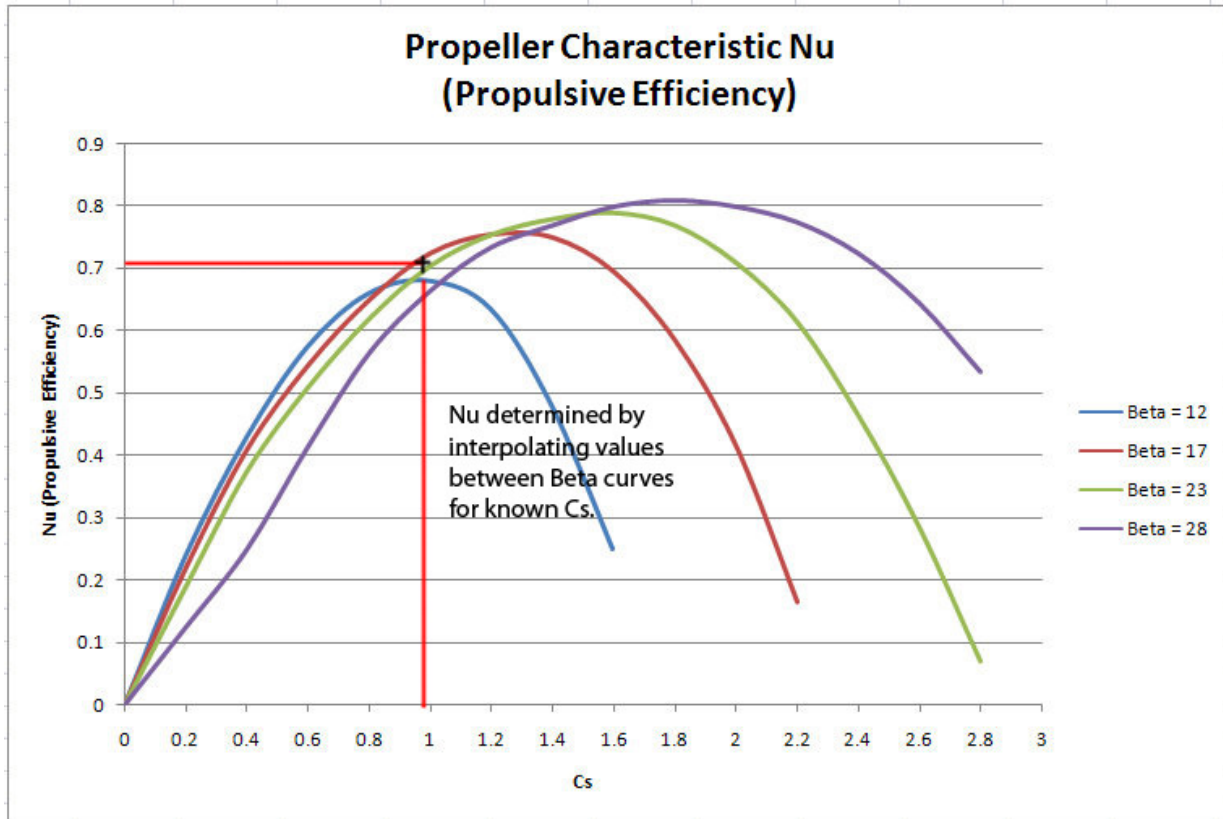
### Beta (Propeller Pitch)

The Lockheed report contains a family of curves on page 25 relating  $V/nD$  to  $C_s$  for various values of the propeller pitch (Beta). Using these curves, we can evaluate  $C_s$  and  $V/nD$  then use them to determine the propeller pitch under a given set of circumstances. To determine the pitch, we simply interpolate between adjacent Beta curves for the  $V/nD$  characteristic.



## η (Propulsive Efficiency)

Now that Cs and Beta have been determined, we can refer to the other family of curves on page 25 to determine η, the propulsive efficiency of the propeller. Computing η, then, is simply a matter of interpolating between these curves.



## Thrust Power Available

Once η has been found, we can convert engine output power ( ) to thrust power ( ) for each engine using the formula:

$$P_{thrust} = \eta \times P_{BHP}$$

The Electra has two engines, so the total thrust available would be

$$P_{thrust} = 2 \times \eta \times P_{BHP}$$

## Rate of Climb

Based on the preceding sections, we can determine the propulsive power required to maintain level flight for any given flight conditions of airspeed, altitude and gross weight. We can also determine thrust power available for the same airspeed and altitude. If the available power is greater than the required power, the aircraft can use the excess power to gain altitude.



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An aircraft gaining altitude at a given rate,  $R_c$ , uses additional power based on its gross weight,  $W$ :

$$P_{climb} = WR_c$$

where  $W$  is the gross weight and  $RC$  is the rate of climb. If weight is expressed in pounds and rate of climb in feet per second, this equation gives power in foot pounds per second. But it’s more convenient to express power in horsepower and rate of climb in feet per minute. Incorporating the appropriate unit conversions, we get

$$P_{climb} = \frac{WR_c}{33000}$$

The Lockheed report doesn’t explicitly discuss rate of climb and its relationship to excess available power but it does give figures on page 16 for initial rate of climb. These values have been found to be consistent with the above formula and the initial gross weight of 16,500 pounds used in the report.

### Fuel Consumption Rate

The specific fuel consumption rate of the Electra’s engines is described in the top left diagram on page 13 as a function of BHP. Two curves are presented, one yielding 4100 mile range, the other 4500. These seem to represent conservative and optimal estimates of achievable engine efficiency. The summary and recommendations section on page 1 of the report has this to say about FCR:

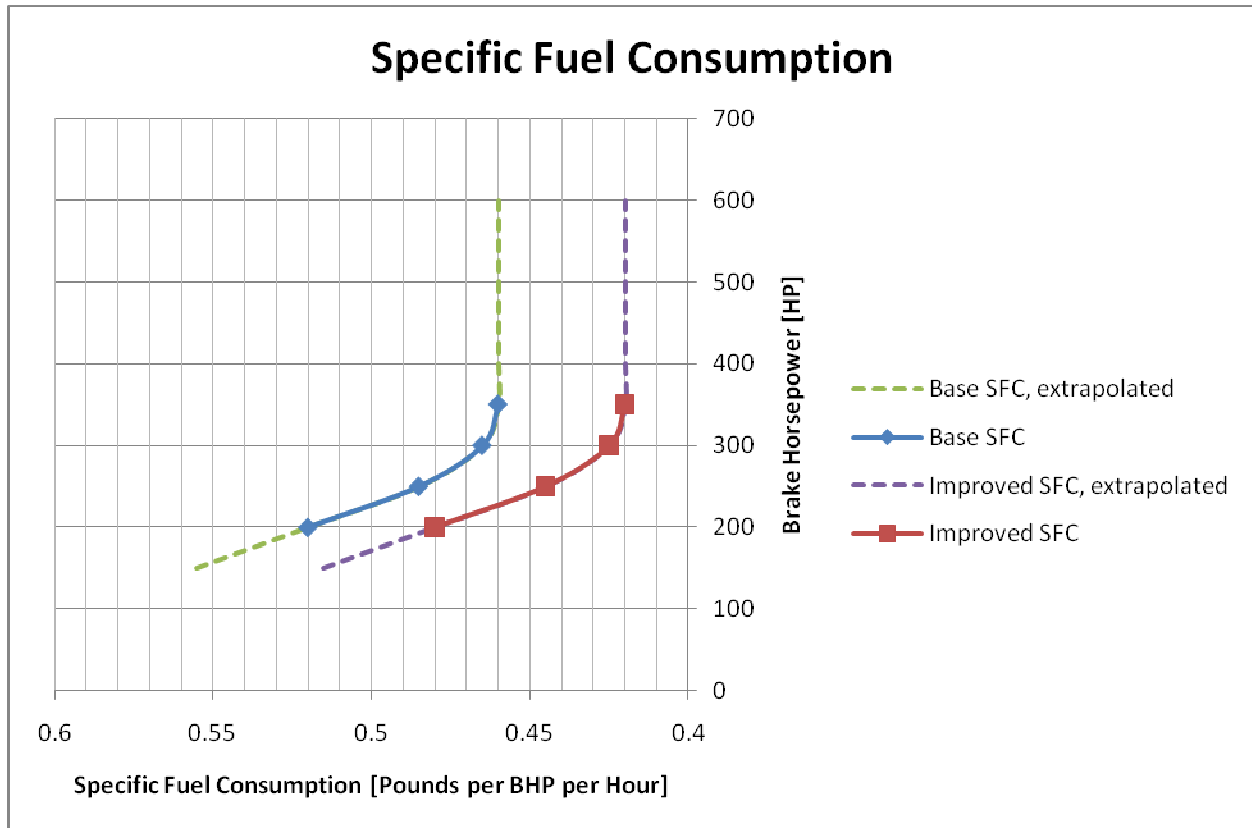
*The complete performance has been computed conservatively based on actual flight test results on Model 10E. Fuel consumption data is based on results which have been obtained in flight with careful mixture control. To get a range of 4500 miles it will be necessary to calibrate the Cambridge Analyzer so that the fuel consumption curve shown on page 13 can be obtained.*

On the other hand, page 20 has this gem:

*Complete data on the fuel consumption of the engine was not available so generalized data on aircooled engines was used.*

So perhaps the “4100-mile range” curve is based on actual flight tests and the more efficient “4500-mile range” curve represents an expected improvement made possible by proper calibration of the Cambridge Gas Analyzers. Or perhaps both of these curves are somewhat hypothetical. In the investigations to follow, we will want to vary the fuel consumption rate to study the effect it has on the maximum range.

These curves only cover the power range from 200 to 350 Horsepower. But other areas of the report describe operating conditions of up to 400 BHP in normal flight and 600 BHP during takeoff. For the purposes of the simulation, we assume that the efficiency curve can be extrapolated to higher BHP values by keeping efficiency constant above 350 BHP. Similarly, the simulation extends the FCR curves down to 150 BHP by extrapolating the curves.



### Computation Procedure

Now that we have all of the formulas developed above, we are ready to describe how they are employed to create a mathematical model of the aircraft.

The form of our mathematical model will be a set of differential equations. A state vector is defined that describes the aircraft’s position, velocity, altitude and fuel load. Then the formulas defined above are used to find the derivatives of each component of the state vector. Once the derivatives are found, it is a relatively straightforward to perform a numerical integration to project the state vector forward in time.

The state vector used by our model consists of five components: distance ( $d$ ), ground speed ( $v_{ground}$ ), altitude ( $h$ ), regular octane fuel load ( $m_{reg}$ ) and high octane fuel load ( $m_{ho}$ ).

In addition to the state vector, there are a number of other parameters and sub-models used in the computations. Headwind ( $v_{wind}$ ) is used when computing airspeed ( $v$ ) from groundspeed ( $v_{ground}$ ). A standard atmosphere model gives an estimate of air density,  $\rho$ , as a function of altitude,  $\rho(h)$ .

The simulation includes a model pilot that is responsible for setting several flight control inputs. Engine power,  $P_{BHP}$ , represents the engine output power (per engine). Propeller speed ( $n$ ), in RPM, represents the setting of the constant speed prop controller. Desired climb rate ( $DR_c$ ) represents the rate of climb the pilot wants to achieve. Finally, the pilot selects one of the two modeled fuel tanks: the regular



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octane fuel or high octane. In the event that the selected tank is empty, the model will use a value of 0 for engine output power,  $P_{BHP}$ , rather than the value specified by the pilot.

Given an arbitrary state vector,  $y$ , at time  $t$ , we can evaluate the time derivative of  $y$  using the following procedure:

compute airspeed,  $v$  :

$$v = v_{ground} + v_{wind} .$$

Compute air density,  $\rho$  , from standard atmosphere model:

$$\rho = \rho (h)$$

Compute dynamic pressure,  $q$ :

$$q = \frac{1}{2} \rho v^2$$

Compute total aircraft mass,  $m$ , by summing fuel masses and aircraft empty mass:

$$m = m_{empty} + m_{fuel} + m_{ho}$$

Multiply mass by gravity to get the total weight of the aircraft:

$$W = mg$$

Compute coefficient of lift,  $C_L$ , at this operating point:

$$C_L = \frac{W}{Aq}$$

Compute coefficient of drag  $C_D$ , from coefficient of lift:

$$e = 0.85 - .0667 \times C_L$$

$$C_D = 0.029 + \frac{C_L^2}{20.75 e}$$

Compute  $C_s$ :

$$C_s = \sqrt[3]{\frac{\rho v^5}{P \omega^2}}$$



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Compute  $V/nD$ :

$$\frac{v}{nD}$$

Lookup Beta as described above.

Lookup Nu as described above.

Compute thrust power, THP:

$$P_{thrust} = 2 \times \eta \times P_{BHP}$$

Compute power required for level flight ( $P_{level}$ ):

$$P_{level} = C_D A q v$$

Compute additional power required for desired rate of climb ( $P_{climb}$ ):

$$P_{climb} = W \times DR_c$$

Compute excess power available for acceleration ( $P_{accel}$ ):

$$P_{accel} = P_{thrust} - P_{level} - P_{climb}$$

It is worth noting at this point that the power required for the desired climb rate may be greater than the available power. In this case acceleration power will be negative resulting in a loss of airspeed. This is a natural consequence of gaining altitude too fast – the aircraft slows down. If, however, the airspeed goes too low, the aircraft will approach the stall condition where  $C_L$  exceeds its maximum. When this condition is approached, the simulation does not allow the airspeed to drop any further and instead reduces climb rate to maintain the minimum airspeed.

```
if (Paccel < 0) and (Cl > 0.9 Clmax)
    Pclimb = Pclimb + Paccel // adding a negative value to Pclimb reduces it.
    Paccel = 0
```

The actual rate of climb, then, is computed from the adjusted climb power:

$$AR_c = \frac{P_{climb}}{W}$$

Finally, we are able to compute the net horizontal force,  $f$ , on the aircraft:

$$f = \frac{P_{accel}}{v}$$

And the resulting acceleration is

$$a = \frac{f}{m}$$



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The fuel consumption rate is a function of engine output power and is given by the curve on page 13 of the Lockheed report:

$$sfc = SFC(BHP)$$

The specific fuel consumption can be converted to a fuel consumption rate by multiplying by the actual BHP settings and number of engines:

$$FCR = 2 \times sfc \times P_{BHP}$$

Finally, we have all the values we need to evaluate the time derivative of the original state vector:

State Component	Derivative	Note
d	$v_{ground}$	Derivative of distance is speed from state vector
$v_{ground}$	a	Derivative of speed is acceleration from preceding computations.
h	$AR_c$	Derivative of altitude is actual rate of climb
$m_{reg}$	FCR if regular fuel tank selected, 0 otherwise.	Derivative of fuel quantity is fuel consumption rate when this tank is selected.
$m_{ho}$	FCR if high octane fuel tank selected, 0 otherwise.	Derivative of fuel quantity is fuel consumption rate when this tank is selected.

### From Mathematical Model to Simulator

At this point, we have mathematical expressions for the derivative of the aircraft’s state vector under an arbitrary set of pilot inputs and flight conditions. Now we are ready to describe the integrator and other software components that can project the flight forward in time. A complete description of the software system is beyond the scope of this paper. So in this section we will only highlight key aspects of the simulation that the reader should understand.

#### Integrator

The simulation uses an integration method called Merson’s method with error estimation and variable step size control. The interested reader is referred to the book “Math Toolkit for Real-Time Programming” by Jack Crenshaw. The integration engine used by this simulation is based directly on chapters 11 and 12 of that book.





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This integration method is very accurate and efficient. It provides an estimation of the error in its final results. When integrating the equations outlined earlier over a flight of 4032 miles, for example, the error estimate is only 37 feet. So the integration itself should not be a significant source of error in the simulation.

### Electra Model

At the heart of the simulation is an object representing the Electra 10E aircraft. This object performs the calculation of the state vector derivative for the integration engine. It also presents an interface to the pilot model through which the pilot can set flight control inputs.

When initialized, the Electra model computes a fuel consumption rate table based on page 13 of the Lockheed report. But since two curves are given in the report, the model must be told which one to use or, more precisely, it must be told how to interpolate between them. So the Electra model defines a parameter called the efficiency metric. If the program sets this parameter to zero, the conservative fuel consumption rate is used. A value of one will cause the more optimistic curve to be used. Values between these two can produce intermediate fuel economy and negative values can even produce worse than expected fuel economy.

### Units

In the Lockheed report, traditional English units are used for all quantities. Furthermore, different units are sometimes used for the same types of quantities. For example, speeds are in miles per hour (ground speed) and in feet per minute (rate of climb.) In order to simplify the physics equations, the software uses SI units and the MKS (meter, kilogram, seconds) system for all quantities. A convention is used that user input and constants in the source code are entered in their original English units and then converted to the appropriate SI units before being stored in variables. Then all computations proceed using SI units. When results are reported from the program, a conversion is made back to the appropriate English units just before the data is formatted for output.

The class, Units, implements methods for performing all the required unit conversions. These conversions should have no impact on the simulation’s accuracy or results.

### Treatment of Airspeed vs Groundspeed

The simulation uses a reference frame associated with the ground. So the state vector’s distance and speed components represent ground distance and ground speed, respectively. The simulation can accommodate a changing headwind. In each integration step, it evaluates the airspeed by adding the headwind to the ground speed.

### Treatment of Fuel Volume, Mass and Weight

Fuel quantities are a potential source of controversy in the field of Earhart investigations. The Lockheed report consistently uses US gallons to refer to fuel quantities. But gasoline changes its density with temperature. The report seems to use a density value of 6 pounds per gallon which corresponds to standard temperature, 58 degrees Fahrenheit.



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But when the Electra was fueled at Lae, New Guinea, temperatures were certainly higher. So the simulation allows the user to input initial fuel loads in gallons at a temperature specified in degrees Fahrenheit. It then converts gallons to pounds using the appropriate density for the specified temperature. Then pounds are converted kilograms before the simulation begins. At various times during the simulation, the remaining fuel quantity is reported in terms of gallons. These are gallons at standard temperature. The initial temperature is not used to adjust these volumes. (And because the initial fuel load is adjusted for temperature before the simulation starts and then reported in gallons at standard temperature, a user may notice an unexpected change in the fuel quantity at the beginning of the simulation.)

### Pilot Model

In the real world, an aircraft relies on a pilot to provide flight inputs to control its trajectory. The simulation uses an analogous object to represent the pilot. The pilot object is responsible for setting the engine power, prop speed and desired climb rate for the aircraft. The RangeTestPilot class implements the behavior recommended in the Lockheed report. It flies the altitude and airspeed profiles given for maximum range using the recommended procedures for climbing and descending.

In the real world, a pilot who wanted to establish a desired airspeed would adjust the throttle and prop speed until the plane was flying at the appropriate airspeed. In the simulation a somewhat different algorithm is used to achieve the same effect. The pilot object can call a method on the Electra asking what throttle and prop speed will yield a desired airspeed, and rate of climb at a given altitude. The Electra performs a binary search over the valid range of engine powers and finds the appropriate power setting. It also finds an appropriate prop speed and the achievable rate of climb for this power setting. The pilot then tells the aircraft to adopt said engine power, prop speed and rate of climb. It’s a little different from the real-world technique but should not make any material difference in the results.

The pilot model can be configured to react in one of two ways to the situation where the regular octane fuel runs out. The plane has two simulated tanks, one with high octane fuel for takeoff, and the other with regular fuel. During takeoff, the high octane tank is selected. Then, after takeoff, the pilot selects regular fuel for the remainder of the flight. When this runs out, she can choose to glide the plane back to the ground or switch to the remaining high octane tank and continue flying. The former behavior corresponds to a pilot forgetting about the extra, high octane fuel on board.

When the plane’s fuel is finally exhausted, the pilot sets flight inputs to perform a glide back to earth. The goal of the glide phase is to minimize the rate of descent. When the aircraft is only a few feet from the ground, the goal changes to minimizing airspeed while holding a constant altitude. So when the flight ends, the aircraft has covered the maximum possible distance and the plane lands at its minimum speed.

### Navigation

At this point in time, the simulation contains no provisions for modeling navigation. The flight is assumed to be in a straight line from beginning to end.



## Modeling the Range Performance of the Electra 10E – Amelia Earhart’s Aircraft

### Reproducing the Range Report’s Results

The range report recommended a specific flight profile on page 7 along with recommendations on procedures for takeoff, ascending, descending and cruise (pp2,3). And it states that these procedures would yield a flight of about 4100 miles in about 26.5 hours with zero headwinds.

The simulation developed here, given the same initial conditions produces a flight of 4034 miles in 28:03 hours.

#### Simulation Parameters

<b>Initial Fuel</b>	1200 gal
<b>Fuel Temp</b>	58F
<b>Headwind</b>	0 mph
<b>Flight Profile</b>	p. 7 of report
<b>Efficiency Metric</b>	0.0

#### Results

	<b>Report Value</b>	<b>Simulation Value</b>
<b>Range</b>	4100 mi	4034 mi
<b>Endurance</b>	26:30 hours	28:03 hours

The difference in range of 66 miles over a 4100 mile flight represents an error of just 1.5%. The difference in estimated time, 28 hours vs 26.5, represents a larger error but is still a difference of only 5%.

### Conclusions

Based on these results, it is fair to conclude that the mathematical model developed in this paper is a faithful analog of the techniques used by Kelly Johnson and W.C. Nelson to analyze the Electra 10E’s maximum range capabilities. Furthermore, the propeller characteristics and lift and drag characteristics that form the basis of this model appear to be based on actual flight test results. Only the specific fuel consumption data seem to be somewhat soft. If we assume that the aerodynamic characteristics of Earhart’s plane are essentially the same as those presented in this paper, then, for some value of the efficiency metric, this computer model should accurately reproduce the performance of her aircraft.



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### References

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